E 2.5 Signals & Linear Systems (1)

Tutorial Sheet 7 – Sampling
Solutions

1. Consider $f(t) = \mathrm{sinc}(kt)$

$$F(\omega) = \frac{\pi}{k}\,\mathrm{rect}\left(\frac{\omega}{2k}\right)$$

$$\int_{-\infty}^{\infty} \mathrm{sinc}^2(kt)\,dt = E\left(f(t)\right)$$ Energy of $f(t)$

∴ Use Parseval's Theorem:–

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\pi^2}{k^2}\left[\mathrm{rect}\left(\frac{\omega}{2k}\right)\right]^2 d\omega$$

$$= \frac{\pi}{2k^2}\int_{-k}^{k} d\omega = \frac{\pi}{k}$$ //

2. a) Bandwidth of $f_1(t)$ is 100 kHz.
∴ Nyquist rate = 200 kHz. //

b) Bandwidth of $f_2(t)$ is 150 kHz.
∴ Nyquist rate = 300 kHz //

c) $f_1^2(t) \Longleftrightarrow \frac{1}{2\pi} F_1(\omega) * F_1(\omega)$.
From width property of convolution, bandwidth of $f_1^2(t)$ is twice bandwidth of $f_1(t)$
∴ Nyquist rate = 400 kHz //

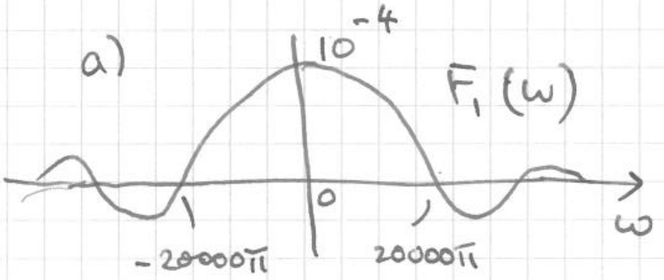
d). Similarly Nyquist rate $f_2^3(t) = 3 \times 300\text{kHz} = 900\text{kHz}$ //

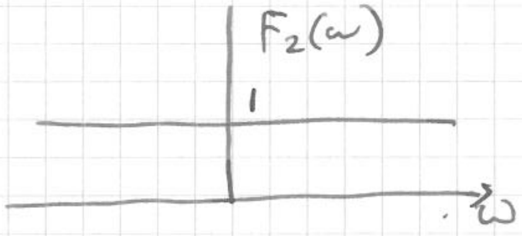
e). Bandwidth of $f_1(t) f_2(t)$ is sum of individual BW.
∴ Nyquist rate = 500 kHz //

3. $F_1(\omega) = 10^{-4} \operatorname{sinc}\left(\frac{\omega}{20000}\right)$

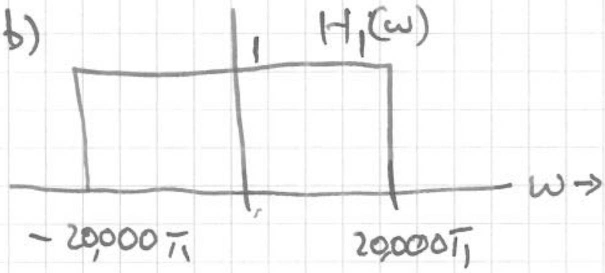
$F_2(\omega) = 1$

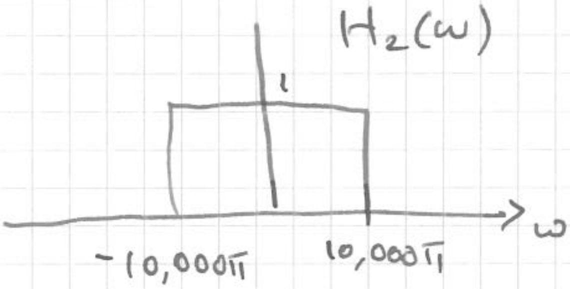
a)



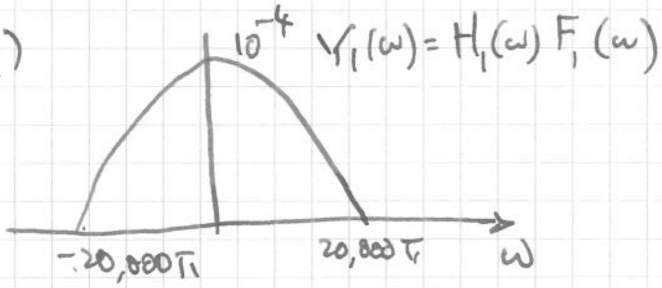


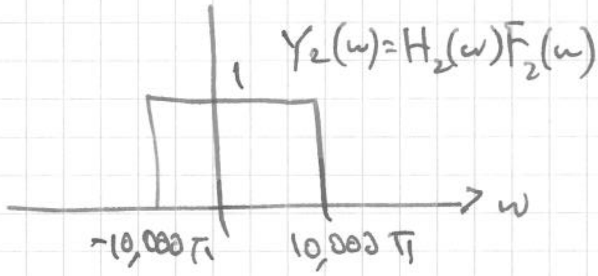
b)





c)





d) BW of $y_1(t)$ = 10kHz
BW of $y_2(t)$ = 5 kHz.
$\therefore$ BW of $y_1(t)\,y_2(t)$ is 15kHz
Nyquist rate = 30kHz.

4. $$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_0^{\infty} e^{-2at}\,dt = \frac{1}{2a}.$$

Better:

~~for~~ use Parseval's theorem

$$X(\omega) = \frac{1}{j\omega + a}.$$

$$\therefore E_x = \frac{1}{\pi}\int_0^{\infty} |X(\omega)|^2\,d\omega$$

$$= \frac{1}{\pi}\int_0^{\infty} \frac{1}{\omega^2 + a^2}\,d\omega$$

$$= \frac{1}{\pi a}\tan^{-1}\frac{\omega}{a}\Big|_0^{\infty} = \frac{1}{2a}.$$

$$99\% \text{ of } E_x = \frac{0.99}{2a}$$

$$\therefore \frac{0.99}{2a} = \frac{1}{\pi}\int_0^{W} \frac{d\omega}{\omega^2 + a^2} = \frac{1}{\pi a}\tan^{-1}\frac{\omega}{a}\Big|_0^{W}$$

$$= \frac{1}{\pi a}\tan^{-1}\frac{W}{a}$$

$$\Rightarrow \frac{0.99\pi}{2} = \tan^{-1}\frac{W}{a}$$

$$\Rightarrow W = 63.66\,a \text{ rads/s}$$

$$= 10.13\,a \text{ Hz}$$ //

5. a) Input = $\delta(t)$

Input to integrator is $\left[\delta(t) - \delta(t-T)\right]$.

$$\therefore h(t) = \int_0^t \left[\delta(\tau) - \delta(\tau - T)\right] d\tau$$

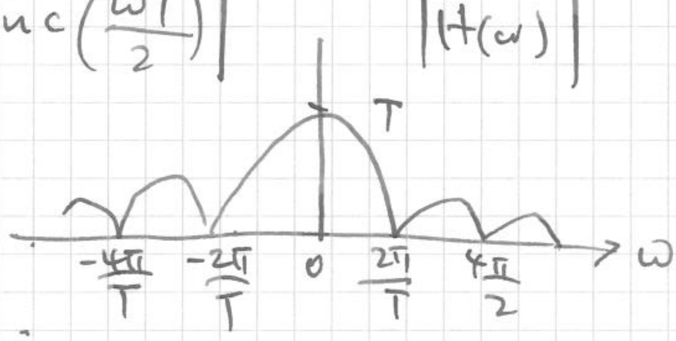
$$= u(t) - u(t-T)$$

$$= \text{rect}\left(\frac{t - \frac{T}{2}}{T}\right)$$

b) Transfer function of cct is FT of $h(t)$

$$\therefore \quad H(\omega) = T \,\text{sinc}\left(\frac{\omega T}{2}\right) e^{-j\omega T/2}$$

$$|H(\omega)| = T \left|\text{sinc}\left(\frac{\omega T}{2}\right)\right|$$



This has lowpass filtering effect. BW of filter $\simeq \frac{1}{T}$ Hz.

c). The impulse response is a rectangular pulse of width T. $\therefore$ When a sampled signal is applied to this, the samples are convolved with this pulse.

$\therefore$